Qualifying Exam Syllabus

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1 Major topic: Algebraic Number Theory (Algebra)

References: Neukirch, *Algebraic Number Theory*, Ch I.1-10, II.1-8,
Cassels & Frohlich, *Algebraic Number Theory*, Ch VI, VII

- **Number Fields:** integrality, norm and trace, Dedekind domains, ideal factorization and class group, lattices and Minkowski bound, Dirichlet's unit theorem
- **Local Theory:** p-adic numbers, completions, valuations and absolute values, extensions of valuations, Hensel's lemma, local and global fields, ramification of extensions
- **Class Field Theory:** adeles and ideles, statements of local and global class field theory, statement of Artin reciprocity, statement of Chebotarev density

2 Major topic: Probability Theory (Probability)

References: Durrett, *Probability: Theory and Examples 5th Ed.*, Chapters 1–5

- **Preliminaries:** σ-algebras, Dynkin's π-λ theorem, independence, Borel–Cantelli lemmas, Kolmogorov's 0-1 law, Kolmogorov's maximal inequality, strong and weak laws of large numbers
- **Central limit theorems:** weak convergence, characteristic functions, tightness, I.I.D. central limit theorem, Lindeberg–Feller central limit theorem
- **Conditioning:** conditional probability and expectation, regular conditional probabilities
- **Martinagles:** stopping times, upcrossing inequality, uniform integrability, A.S. convergence, Doob's decomposition, Doob's inequality, L^p convergence, L^1 convergence, reverse martingale convergence, optional stopping theorem, Wald's identity
- **Markov chains:** countable state space, stationary measures, convergence theorems, recurrence and transience, asymptotic behavior

3 Minor topic: Complex Analysis (Analysis)

References: Stein and Shakarchi, *Complex Analysis*, Chapters 1-3

- **Complex functions:** holomorphic, meromorphic, Cauchy-Riemann equations, Liouville's theorem, Taylor and Laurent series
- **Complex integration:** Cauchy's theorem, Cauchy's integral formula, residue theorem, argument principle, Rouché's theorem, Morera's Theorem, maximum modulus principle
- **Fundamental Theorem of Algebra:** Statement and proof